CSE 125
Discrete Mathematics

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Predicates

- “x > 3”
- “x = y + 3”
- “x + y = z”

Predicates

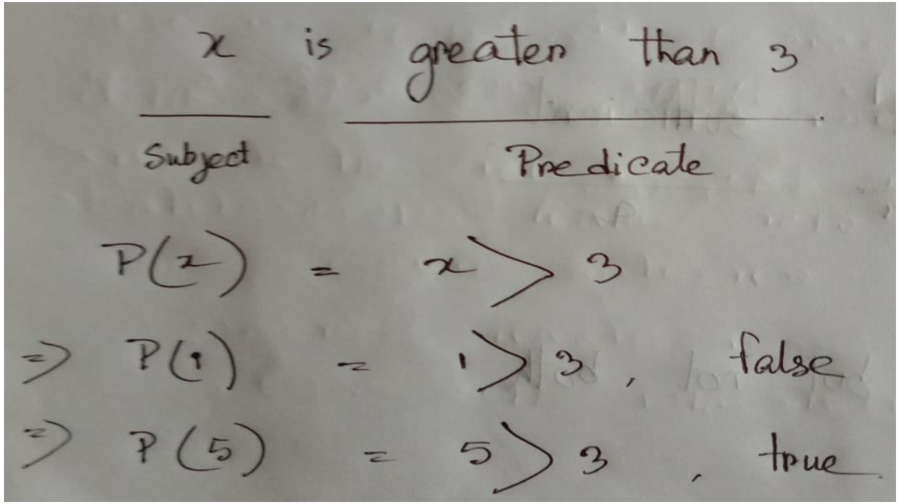
- Express the meaning of a wide range of statements in mathematics and computer science
- Permit us to reason and explore relationships between objects.

Predicate

"x is greater than 3"

- The first part, the variable x, is the subject.
- The second part—the predicate.

Predicate


x is greater than 3
Subject
Predicate
P(x) = x > 3
=> P(1) = 1 > 3 , false
=> P(5) = 5 > 3 , true

Predicates of Multiple Variables

$x + y > 3$, $P(x, y)$

$x + y = z$, $P(x, y, z)$

Problem

Let P (x) denote the statement "x > 3." What are the truth values of P (4) and P (2)?

Solution: For P (4), the statement is "4 > 3," is true.
For P (2), the statement is "2 > 3," is false.

Problem

Let Q(x, y) denote the statement “x = y + 3.” What are the truth values of the propositions Q(1, 2) and Q(3, 0)?

Solution: For, Q(1, 2) the statement is “1 = 2 + 3,” false.

For Q(3, 0) the statement is “3 = 0 + 3,” true

Quantifiers

- Quantification expresses the extent to which a predicate is true over a range of elements
- The area of logic that deals with predicates and quantifiers is called the predicate calculus.

Category of Quantifiers

- **Universal quantification**: A predicate is true for every element under consideration
- **Existential quantification**: There is one or more element under consideration for which the predicate is true.

Definition

- The universal quantification of $P(x)$ is the statement "$P(x)$ for all values of x in the domain."
- The notation $\forall x P(x)$ denotes the universal quantification of $P(x)$. Here $\forall$ is called the universal quantifier. We read $\forall x P(x)$ as "for all x $P(x)$" or "for every x $P(x)$."

Expressing Universal Quantifiers

- “for all”
- “for every”
- “all of”
- “for each”

Expressing Universal Quantifiers

- “given any”
- “for arbitrary”
- “for each”
- “for any”

Problem

Let $P(x)$ be the statement "$x + 1 > x$." What is the truth value of the quantification $\forall x P(x)$, where the domain consists of all real numbers?

Solution: Because $P(x)$ is true for all real numbers x, the quantification $\forall x P(x)$ is true.

Problem

Let $Q(x)$ = "$x < 2$." What is the truth value of the quantification $\forall x Q(x)$, where the domain consists of all real numbers?

Solution: $Q(x)$ is not true for every real number x, because, for instance, $Q(3)$ is false. That is, $x = 3$ is a counterexample for the statement $\forall x Q(x)$. Thus $\forall x Q(x)$ is false.

THE EXISTENTIAL QUANTIFIER

- The existential quantification of P (x) is the proposition "There exists an element x in the domain such that P (x)."
- ∃xP (x) us the notation for the existential quantification of P (x). ∃ is called the existential quantifier

Expressing Existential Qualifier

- “There is an x such that P (x),”
- “There is at least one x such that P (x),”
- “For some x P (x).”

Problem

Let P (x) denote the statement “x > 3.” What is the truth value of the quantification ∃xP (x), where the domain consists of all real numbers?

Solution: Because “x > 3” is sometimes true—for instance, when x = 4—the existential quantification of P (x), which is ∃xP (x), is true

Problem

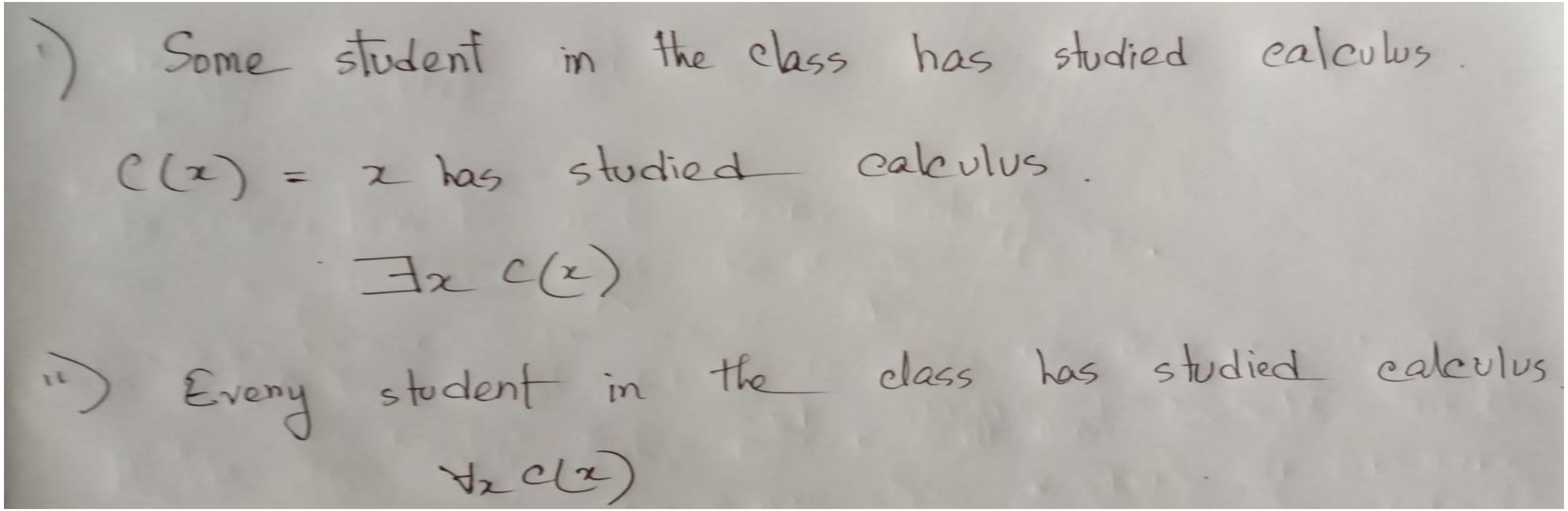
Let $Q(x)$ denote the statement "$x = x + 1$."What is the truth value of the quantification$\exists x Q(x)$, where the domain consists of all real numbers?

Solution: Because $Q(x)$ is false for every real number x, the existential quantification of $Q(x)$, which is $\exists x Q(x)$, is false

Summarization

TABLE 1 Quantifiers.		
Statement	*When True?*	*When False?*
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

Logical Sentences Using Predicators



i) Some student in the class has studied calculus.

$C(x)$ = x has studied calculus.

$\exists x\, C(x)$

ii) Every student in the class has studied calculus.

$\forall x\, C(x)$

Logical Sentences Using Predicators

1) $S(x)$ = x is a student in this class.

$C(x)$ = x has studied calculus.

$$\forall x \, (S(x) \longrightarrow C(x))$$

For all x, if x is a student in this class,

then x has studied et calculus.

Logical Sentences Using Predicators

11) $S(x)$ = x is a student in this class

$M(x)$ = x has visited Mexico

$$\exists x \left(S(x) \longrightarrow M(x) \right)$$

There is a person x having the properties that x is a student in this class and x has visited Mexico

Negating Quantified Expressions

- $\neg \forall x P(x) \equiv \exists x \neg P(x)$
- $\neg \exists x Q(x) \equiv \forall x \neg Q(x)$

Negating Quantified Expressions

1) Every student in your class has taken a course in calculus.

$C(x)$ = x has taken a course in calculus

$\forall x \, C(x)$

Negating Quantified Expressions

$$\equiv \neg(\forall x \, C(x))$$

$$\equiv \exists x \, \neg C(x)$$

There is a student in your class, who hasn't taken a course in calculus.

Negating Quantified Expressions

ii) There is a student in this class who has taken a course in calculus.

$c(x)$ = x has taken a course in calculus.

$\exists_x c(x)$

$= \neg(\exists_x c(x))$

$= \forall_x \neg c(x)$

= Every student in this class has not taken calculus

De Morgan's Laws for Quantifiers

TABLE 2 De Morgan's Laws for Quantifiers.

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

Nested Quantifiers

- $\forall x \exists y(x + y = 0)$
- $\forall x \forall y(x + y = y + x)$
- $\forall x \forall y \forall z(x + (y + z) = (x + y) + z)$
- $\forall x \forall y((x > 0) \land (y < 0) \rightarrow (xy < 0))$

Problem

- Translate into English the statement $\forall x \forall y((x > 0) \wedge (y < 0) \rightarrow (xy < 0))$, where the domain are real numbers.

Solution

- For every real number x and y, if $x > 0$ and $y < 0$, then $xy < 0$.
- "The product of a positive real number and a negative real number is always a negative real number."

Problem

“There is a real number y such that for every real number x, Q(x, y).”

Solution: $\forall x \exists y Q(x, y)$

Quantification of Two Variables

TABLE 1 Quantifications of Two Variables.

Statement	*When True?*	*When False?*
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair x, y.	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y.
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y.